Section 10.7
Probability

The Weather Outside is Frightful!

Have you ever thought about the chances of being hit by lightning, caught in a tornado, hurricane, or some other major weather event?

In one of the application exercises in this section, mathematicians, meteorologists, and you will team up to determine such probabilities.

Objective #1: Compute empirical probability.

✔ *Solved Problem #1*

1. Use the data in the table to find the probabilities.

Mammography Screening on 100,000 U.S. Women, Ages 40 to 50	Breast Cancer	No Breast Cancer
Positive Mammogram	720	6944
Negative Mammogram	80	92,256

1a. Find the probability that a woman aged 40 to 50 has a positive mammogram.

The probability of having a positive mammogram is the number of women with a positive mammogram divided by the total number of women.

$$P(\text{positive mammogram}) = \frac{720+6944}{100{,}000} = \frac{7664}{100{,}000} \approx 0.077$$

✎ *Pencil Problem #1* ✎

1. The table shows the distribution, by marital status and gender, of the 242 million Americans ages 18 or older. Use the table to find the probabilities.

	Never Married	Married	Widowed	Divorced
Male	40	65	3	10
Female	34	65	11	14

1a. If one person is randomly selected from the population described in the table, find the probability, to the nearest hundredth, that the person is divorced.

1b. Among women with positive mammograms, find the probability of having breast cancer.
(Use the data in the table on the previous page)

To find the probability of breast cancer among women with positive mammograms, restrict the data to women with positive mammograms:

Mammography Screening on 100,000 U.S. Women, Ages 40 to 50	Breast Cancer	No Breast Cancer
Positive Mammogram	720	6944

$$P(\text{breast cancer}) = \frac{720}{720+6944} = \frac{720}{7664} \approx 0.094$$

1b. Among those who are divorced, find the probability of selecting a woman.
(Use the data in the table on the previous page)

Objective #2: Compute theoretical probability.

✔ *Solved Problem #2*

2a. A die is rolled. Find the probability of getting a number greater than 4.

Two of the six numbers, 5 and 6, are greater than 4.

$$P(\text{greater than } 4) = \frac{2}{6} = \frac{1}{3}$$

2b. The original Florida LOTTO was set up so that each player chose six different numbers from 1 to 49. With one LOTTO ticket, what was the probability of winning the top cash prize? Express the answer as a fraction and as a decimal correct to ten places.

$$_{49}C_6 = \frac{49!}{(49-6)!6!} = \frac{49!}{43!6!} = \frac{49\times48\times47\times46\times45\times44\times\cancel{43!}}{\cancel{43!}\times6\times5\times4\times3\times2\times1} = 13{,}983{,}816$$

$$P(\text{winning LOTTO}) = \frac{1}{13{,}983{,}816} \approx 0.0000000715$$

Pencil Problem #2

2a. A die is rolled. Find the probability of getting a 4.

2b. To play the California lottery, a person has to correctly select 6 out of 51 numbers. If you pick six numbers that are the same as the ones drawn by the lottery, you win. What is the probability that a person with one combination of six numbers will win?

Objective #3: Find the probability that an event will not occur.

✔ *Solved Problem #3*

3. Of the 7000 million people in the world, 550 million live in North America. If one person is randomly selected from the world population, find the probability that the person does not live in North America.

$$P(\text{not North America}) = 1 - P(\text{North America})$$
$$= 1 - \frac{550}{7000}$$
$$= \frac{6450}{7000}$$
$$= \frac{129}{140}$$

Pencil Problem #3

3. If you are dealt one card from a 52-card deck, find the probability that you are *not* dealt a king.

Objective #4: Find the probability of one event or a second event occurring.

✔ *Solved Problem #4*

4a. If you roll a single, six-sided die, what is the probability of getting either a 4 or a 5?

These events are mutually exclusive.
Thus, add their individual probabilities.

$$P(4 \text{ or } 5) = P(4) + P(5)$$
$$= \frac{1}{6} + \frac{1}{6}$$
$$= \frac{2}{6}$$
$$= \frac{1}{3}$$

4b. Each number, 1 through 8, is written on slips of paper and placed in a hat. If one number is selected at random, find the probability that the number selected will be an odd number or a number less than 5.

These events are *not* mutually exclusive. Thus, use the formula $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

$$P(\text{odd or less than 5})$$
$$= P(\text{odd}) + P(\text{less than 5}) - P(\text{odd and less than 5})$$
$$= \frac{4}{8} + \frac{4}{8} - \frac{2}{8}$$
$$= \frac{6}{8}$$
$$= \frac{3}{4}$$

Pencil Problem #4

4a. If you are dealt one card from a 52-card deck, find the probability that you are dealt a 2 or a 3.

4b. Each number, 1 through 8, is written on slips of paper and placed in a hat. If one number is selected at random, find the probability that the number selected will be an odd number or a number less than 6.

Objective #5: Find the probability of one event and a second event occurring.

✔ Solved Problem #5

5a. On a roulette wheel, the ball can land with equal probability on any one of the 38 numbered slots, two of which are green. Find the probability of green occurring on two consecutive plays.

The events are independent.
Thus, use the formula $P(A \text{ and } B) = P(A) \times P(B)$.

$$\begin{aligned} P(\text{green and green}) &= P(\text{green}) \times P(\text{green}) \\ &= \frac{2}{38} \times \frac{2}{38} \\ &= \frac{1}{361} \\ &\approx 0.00277 \end{aligned}$$

5b. Find the probability of a family having four boys in a row.

The events are independent.
Thus, multiply their probabilities.

$$\begin{aligned} P(\text{4 boys in a row}) &= P(\text{boy and boy and boy and boy}) \\ &= P(\text{boy}) \times P(\text{boy}) \times P(\text{boy}) \times P(\text{boy}) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{16} \end{aligned}$$

Pencil Problem #5

5a. A single die is rolled twice. Find the probability of rolling a 2 the first time and a 3 the second time.

5b. If you toss a fair coin six times, what is the probability of getting all heads?