Section 10.6
Counting Principles, Permutations, and Combinations

I Have NOTHING to Wear!

On many mornings, we feel quite limited on the fashion statement we desire to make that day. But the truth is we usually have more clothing options than we might think. If we consider how the various components of our outfit can be mixed and matched, the number of unique outfits can be difficult to count.

Attempting to count each possibility one-by-one can be daunting in many situations.

In this section of the textbook, we will use organized mathematical methods and formulas that will allow us to count more quickly and accurately than the 1, 2, 3, 4, 5, 6, … method.

Objective #1: Use the Fundamental Counting Principle.

✔ *Solved Problem #1*

1a. A pizza can be ordered with three choices of size (small, medium, or large), four choices of crust (thin, thick, crispy, or regular), and six choices of toppings (ground beef, sausage, pepperoni, bacon, mushrooms, or onions). How many different one-topping pizzas can be ordered?

Multiply the number of choices for each of the three decisions:

$$\underline{\text{Size}}: \ \underline{\text{Crust}}: \ \underline{\text{Topping}}:$$
$$3 \times 4 \times 6 = 72$$

72 different one-topping pizzas can be ordered.

1b. License plates in a particular state display two letters followed by three numbers, such as AT-887 or BB-013. How many different license plates can be manufactured?

Multiply the number of choices for each of the letters and each of the digits:

$$\underline{\text{Letter 1}} \ \underline{\text{Letter 2}} \ \underline{\text{Digit 1}} \ \underline{\text{Digit 2}} \ \underline{\text{Digit 3}}$$
$$26 \times 26 \times 10 \times 10 \times 10 = 676{,}000$$

676,000 different license plates can be manufactured.

Pencil Problem #1

1a. An ice cream store sells two drinks (sodas or milk shakes), in four sizes (small, medium, large, or jumbo), and five flavors (vanilla, strawberry, chocolate, coffee, or pistachio). In how many ways can a customer order a drink?

1b. You are taking a multiple-choice test that has five questions. Each of the questions has three answer choices, with one correct answer per question. If you select one of these three choices for each question and leave nothing blank, in how many ways can you answer the questions?

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Objective #2: Use the permutations formula.

✔ *Solved Problem #2*

2a. A corporation has seven members on its board of directors. In how many different ways can it elect a president, vice-president, secretary, and treasurer?

The corporation is choosing 4 officers from a group of 7 people. The order in which the officers are chosen matters because the president, vice-president, secretary, and treasurer each have different responsibilities. Thus, we are looking for the number of permutations of 7 things taken 4 at a time.

$${}_7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = 840$$

There are 840 ways of filling the four offices.

✎ *Pencil Problem #2*

2a. Using 15 flavors of ice cream, how many cones with three different flavors can you create if it is important to you which flavor goes on the top, middle, and bottom?

2b. In how many ways can 6 books be lined up along a shelf?

Because you are using all six of your books in every possible arrangement, you are arranging 6 books from a group of 6 books. Thus, we are looking for the number of permutations of 6 things taken 6 at a time.

$${}_6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{6\times5\times4\times3\times2\times1}{1} = 720$$

There are 720 ways the 6 books can be lined up along the shelf.

2b. What is the number of permutations of 8 things taken 0 at a time?

Objective #3: Distinguish between permutation problems and combination problems.

✔ *Solved Problem #3*

3a. Determine if the question involves combinations or permutations. (Do *not* solve the problem.)

How many ways can you select 6 free DVDs from a list of 200 DVDs?

The order in which the DVDs are selected does not matter.

Thus, this problem involves combinations.

✎ *Pencil Problem #3*

3a. Determine if the question involves combinations or permutations. (Do *not* solve the problem.)

A medical researcher needs 6 people to test the effectiveness of an experimental drug. If 13 people have volunteered for the test, in how many ways can 6 people be selected?

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3b. Determine if the question involves combinations or permutations. (Do *not* solve the problem.)

In a race in which there are 50 runners and no ties, in how many ways can the first three finishers come in?

The order in which the runners finish does matter.

Thus, this problem involves permutations.

3b. Determine if the question involves combinations or permutations. (Do *not* solve the problem.)

How many different four-letter passwords can be formed from the letters A, B, C, D, E, F, and G if no repetition of letters is allowed?

Objective #4: Use the combinations formula.

✔ *Solved Problem #4*

4a. From a group of 10 physicians, in how many ways can four people be selected to attend a conference on acupuncture?

The order in which the four people are selected does not matter. This is a problem of selecting 4 people from a group of 10 people. We are looking for the number of combinations of 10 things taken 4 at a time.

$$
\begin{aligned}
{}_{10}C_4 &= \frac{10!}{(10-4)!4!} \\
&= \frac{10!}{6!4!} \\
&= \frac{10\times 9\times 8\times 7\times 6!}{6!\times 4\times 3\times 2\times 1} \\
&= 210
\end{aligned}
$$

The four attendees can be selected in 210 different ways.

Pencil Problem #4

4a. An election ballot asks voters to select three city commissioners from a group of six candidates. In how many ways can this be done?

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4b. How many different 4-card hands can be dealt from a deck that has 16 different cards?

Because the order in which the 4 cards are dealt does not matter, this is a problem involving combinations. We are looking for the number of combinations of 16 cards drawn 4 at a time.

$$
\begin{aligned}
{}_{16}C_4 &= \frac{16!}{(16-4)!4!} \\
&= \frac{16!}{12!4!} \\
&= \frac{16\times15\times14\times13\times12!}{12!\times4\times3\times2\times1} \\
&= \frac{16\times15\times14\times13\times\cancel{12!}}{\cancel{12!}\times4\times3\times2\times1} \\
&= \frac{16\times15\times14\times13}{4\times3\times2\times1} \\
&= 1820
\end{aligned}
$$

There are 1820 different 4-card hands.

4b. You volunteer to help drive children at a charity event to the zoo, but you can fit only 8 of the 17 children present in your van. How many different groups of 8 children can you drive?

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