Section 10.3
Geometric Sequences and Series

How Much Will You End Up With?

Suppose you are 24 and you have just landed a job!

You decide that you can save for retirement by putting aside \$80 per month into an account which pays 5% compounded monthly.

What will the account balance be when you reach age 65?

In this section, several applications will deal with money and we will apply geometric sequences and series to find answers to a variety of financial questions.

Objective #1: Find the common ratio of a geometric sequence.

✔ *Solved Problem #1*

1. True or False: The sequence
$6, -12, 24, -48, 96, \ldots$
is an example of a geometric sequence.

True. Each term after the first term is -2 times the previous term. The common ration is -2.

Pencil Problem #1

1. True or False: The sequence
$2, 6, 24, 120, \ldots$
is an example of a geometric sequence.

Objective #2: Write terms of a geometric sequence.

✔ *Solved Problem #2*

2. Write the first six terms of the geometric sequence with first term 12 and common ratio $\frac{1}{2}$.

$a_1 = 12,\ r = \dfrac{1}{2}$

$a_2 = 12\left(\frac{1}{2}\right)^1 = 6$

$a_3 = 12\left(\frac{1}{2}\right)^2 = \frac{12}{4} = 3$

$a_4 = 12\left(\frac{1}{2}\right)^3 = \frac{12}{8} = \frac{3}{2}$

$a_5 = 12\left(\frac{1}{2}\right)^4 = \frac{12}{16} = \frac{3}{4}$

$a_6 = 12\left(\frac{1}{2}\right)^5 = \frac{12}{32} = \frac{3}{8}$

The first six terms are $12, 6, 3, \frac{3}{2}, \frac{3}{4}, \text{and } \frac{3}{8}$.

Pencil Problem #2

2. Write the first five terms of the geometric sequence with first term 5 and common ratio 3.

Objective #3: Use the formula for the general term of a geometric sequence.

✔ *Solved Problem #3*

3a. Find the seventh term of the geometric sequence whose first term is 5 and whose common ratio is -3.

$a_1 = 5,\ r = -3$

$a_n = a_1 r^{n-1}$

$a_7 = 5(-3)^{7-1}$

$= 5(-3)^6$

$= 5(729)$

$= 3645$

The seventh term is 3645.

Pencil Problem #3

3a. Find the eighth term of the geometric sequence whose first term is 6 and whose common ratio is 2.

3b. Write the general term for the geometric sequence: $3, 6, 12, 24, 48, \ldots$

Then use the formula for the general term to find the eighth term.

$r = \frac{6}{3} = 2,\ a_1 = 3$

Formula for the general term:

$a_n = a_1 (r)^{n-1}$

$a_n = 3(2)^{n-1}$

Find the eighth term:

$a_n = 3(2)^{n-1}$

$a_8 = 3(2)^{8-1}$

$= 3(2)^7$

$= 3(128)$

$= 384$

The eighth term is 384.

3b. Write the general term for the geometric sequence: $3, 12, 48, 192, \ldots$

Then use the formula for the general term to find the seventh term.

Objective #4: Use the formula for the sum of the first *n* terms of a geometric sequence.

✔ *Solved Problem #4*

4a. Find the sum of the first nine terms of the geometric sequence: $2, -6, 18, -54, \ldots$

$a_1 = 2,\ r = \dfrac{-6}{2} = -3$

$S_n = \dfrac{a_1(1 - r^n)}{1 - r}$

$S_9 = \dfrac{2\left(1 - (-3)^9\right)}{1 - (-3)} = \dfrac{2(19,684)}{4} = 9842$

The sum of the first nine terms is 9842.

Pencil Problem #4

4a. Find the sum of the first 11 terms of the geometric sequence: $3, -6, 12, -24, \ldots$

4b. Find the following sum: $\sum_{i=1}^{8} 2 \times 3^i$.

$a_1 = 2 \times (3)^1 = 6,\ r = 3$

$S_n = \dfrac{a_1(1 - r^n)}{1 - r}$

$S_8 = \dfrac{6\left(1 - 3^8\right)}{1 - 3} = \dfrac{6(-6560)}{-2} = 19,680$

Thus, $\sum_{i=1}^{8} 2 \times 3^i = 19,680.$

4b. Find the following sum: $\sum_{i=1}^{10} 5 \times 2^i$.

4c. A job pays a salary of $30,000 the first year. During the next 29 years, the salary increases by 6% each year. What is the total lifetime salary over the 30-year period? Round to the nearest dollar.

$a_1 = 30,000,\ r = 1.06$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_{30} = \frac{30,000\left(1 - (1.06)^{30}\right)}{1 - 1.06} \approx 2,371,746$$

The total lifetime salary is $2,371,746.

4c. A job pays a salary of $24,000 the first year. During the next 19 years, the salary increases by 5% each year. What is the total lifetime salary over the 20-year period? Round to the nearest dollar.

Objective #5: Find the value of an annuity.

✔ *Solved Problem #5*

5. At age 30, to save for retirement, you decide to deposit $100 at the end of each month into an IRA that pays 9.5% compounded monthly. Find how much will you have in the IRA when you retire at age 65 and find how much is interest.

$$A = \frac{P\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}}$$

$P = 100,\ r = 0.095,\ n = 12,\ t = 35$

$$A = \frac{100\left[\left(1 + \frac{0.095}{12}\right)^{12 \cdot 35} - 1\right]}{\frac{0.095}{12}} \approx 333,946$$

The value of the IRA will be $333,946.

Find the interest:

Interest = Value of IRA - Total deposits

$\approx \$333,946 - \$100 \cdot 12 \cdot 35$

$\approx \$333,946 - \$42,000$

$\approx \$291,946$

✎ *Pencil Problem #5*

5. At age 25, to save for retirement, you decide to deposit $50 at the end of each month into an IRA that pays 5.5% compounded monthly. Find how much will you have in the IRA when you retire at age 65 and find how much is interest.

Objective #6: Use the formula for the sum of an infinite geometric series.

✔ *Solved Problem #6*

6a. Find the sum of the infinite geometric series:

$$3+2+\frac{4}{3}+\frac{8}{9}+\cdots$$

$a_1 = 3,\ r = \frac{2}{3}$

$$S = \frac{a_1}{1-r}$$

$$S = \frac{3}{1-\frac{2}{3}}$$

$$= \frac{3}{\frac{1}{3}}$$

$$= 9$$

The sum of this infinite geometric series is 9.

✎ *Pencil Problem #6* ✎

6a. Find the sum of the infinite geometric series:

$$3+\frac{3}{4}+\frac{3}{4^2}+\frac{3}{4^3}+\cdots$$

6b. Express $0.\overline{9}$ as a fraction in lowest terms.

$$0.\overline{9} = 0.9999\cdots = \frac{9}{10}+\frac{9}{100}+\frac{9}{1000}+\cdots$$

$a_1 = \frac{9}{10}, r = \frac{1}{10}$

$$S = \frac{a_1}{1-r}$$

$$S = \frac{\frac{9}{10}}{1-\frac{1}{10}}$$

$$= \frac{\frac{9}{10}}{\frac{9}{10}}$$

$$= 1$$

An equivalent fraction for $0.\overline{9}$ is 1.

6b. Express $0.\overline{5}$ as a fraction in lowest terms.

