Section 10.2
Arithmetic Sequences

IT'S A FULL THEATER TONIGHT!

Some theaters have the same number of seats in each row. But other theaters are more fan-shaped.

In this section of the textbook, we will encounter such a fan-shaped theater, and we will use the techniques of this section to quickly determine the total number of seats without actually adding the number in each row.

Objective #1: Find the common difference for an arithmetic sequence.

✔ *Solved Problem #1*

1. True or false: An arithmetic sequence is a sequence in which each term after the first differs from the preceding term by a constant amount.

true

✎ *Pencil Problem #1* ✎

1. True or false: In an arithmetic sequence, each term after the first term can be obtained by adding the common difference to the preceding term.

Objective #2: Write terms of an arithmetic sequence.

✔ *Solved Problem #2*

2. Write the first six terms of the arithmetic sequence with first term 100 and common difference -30.

$a_1 = 100$

$a_2 = 100 + (-30) = 70$

$a_3 = 70 + (-30) = 40$

$a_4 = 40 + (-30) = 10$

$a_5 = 10 + (-30) = -20$

$a_6 = -20 + (-30) = -50$

✎ *Pencil Problem #2* ✎

2. Write the first six terms of the arithmetic sequence with first term -7 and common difference 4.

Objective #3: Use the formula for the general term of an arithmetic sequence.

✔ *Solved Problem #3*

3a. Find the ninth term of the arithmetic sequence whose first term is 6 and whose common difference is -5.

$a_1 = 6,\ d = -5$

To find the ninth term, a_9, replace n in the formula with 9, replace a_1 with 6, and replace d with –5.

$$\begin{aligned} a_n &= a_1 + (n-1)d \\ a_9 &= 6 + (9-1)(-5) \\ &= 6 + 8(-5) \\ &= 6 + (-40) \\ &= -34 \end{aligned}$$

3b. In 2010, 16% of the U.S. population was Latino. On average, this is projected to increase by approximately 0.35% per year. Write a formula for the *n*th term of the arithmetic sequence that describes the percentage of the U.S. population that will be Latino n years after 2009.

$$\begin{aligned} a_n &= a_1 + (n-1)d \\ &= 16 + (n-1)0.35 \\ &= 0.35n + 15.65 \end{aligned}$$

3c. Use the result from the previous problem to project the percentage of the U.S. population that will be Latino in 2030.

2030 is 21 years after 2009.

$$\begin{aligned} a_n &= 0.35n + 15.65 \\ a_{20} &= 0.35(21) + 15.65 = 23 \end{aligned}$$

In 2030, 23% of the U.S. population is projected to be Latino.

✎ *Pencil Problem #3* ✐

3a. Find the 50th term of the arithmetic sequence whose first term is 7 and whose common difference is 5.

3b. In 1970, 11.0% of Americans ages 25 and older had completed four years of college or more. On average, this percentage has increased by approximately 0.5 each year. Write a formula for the *n*th term of the arithmetic sequence that models the percentage of Americans ages 25 and older who had or will have completed four years of college or more n years after 1969.

3c. Use the result from the previous problem to project the percentage of Americans ages 25 and older who will have completed four years of college or more by 2019.

Objective #4: Use the formula for the sum of the first *n* terms of an arithmetic sequence.

✔ *Solved Problem #4*

4a. Find the sum of the first 15 terms of the arithmetic sequence: 3, 6, 9, 12, ...

To find the sum of the first 15 terms, S_{15}, replace *n* in the formula with 15.

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$S_{15} = \frac{15}{2}(a_1 + a_{15})$$

Use the formula for the general term of a sequence to find a_{15}. The common difference, *d*, is 3, and the first term, a_1, is 3.

$$\begin{aligned} a_n &= a_1 + (n-1)d \\ a_{15} &= 3 + (15-1)(3) \\ &= 3 + 14(3) \\ &= 3 + 42 \\ &= 45 \end{aligned}$$

Thus, $S_{15} = \frac{15}{2}(3+45) = \frac{15}{2}(48) = 360$.

Pencil Problem #4

4a. Find the sum of the first 50 terms of the arithmetic sequence: $-10, -6, -2, 2, \ldots$

Solved Problem #4

4b. Find the following sum: $\sum_{i=1}^{30}(6i-11)$.

$$\begin{aligned} &\sum_{i=1}^{30}(6i-11) \\ &= (6\cdot 1 - 11) + (6\cdot 2 - 11) + (6\cdot 3 - 11) + \ldots + (6\cdot 30 - 11) \\ &= -5 + 1 + 7 + \ldots + 169 \end{aligned}$$

The first term, a_1, is –5.
The common difference, *d*, is $1-(-5)=6$.
The last term, a_{30}, is 169.

$$\begin{aligned} S_n &= \tfrac{n}{2}(a_1 + a_n) \\ S_{30} &= \tfrac{30}{2}(-5+169) \\ &= 15(164) \\ &= 2460 \end{aligned}$$

Thus, $\sum_{i=1}^{30}(6i-11) = 2460$

Pencil Problem #4

4b. Find the following sum: $\sum_{i=1}^{100} 4i$.

4c. The model $a_n = 1800n + 64,130$ describes yearly adult residential community costs n years after 2013. How much would it cost for the adult residential community for a ten-year period beginning in 2014?

$$a_n = 1800n + 64,130$$
$$a_1 = 1800(1) + 64,130 = 65,930$$
$$a_{10} = 1800(10) + 64,130 = 82,130$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$\begin{aligned} S_{10} &= \frac{10}{2}(a_1 + a_{10}) \\ &= 5(65,930 + 82,130) \\ &= 5(148,060) \\ &= \$740,300 \end{aligned}$$

It would cost $740,300 for the ten-year period beginning in 2014.

4c. A section in a stadium has 20 seats in the first row, 23 seats in the second row, increasing by 3 seats each row for a total of 38 rows. How many seats are in this section of the stadium?