Section 10.1
Sequences and Summation Notation

Bees, Trees, and Piano Keys !

What can those three things possibly have in common?

In this section, we will study sequences. One amazing example is called the Fibonacci sequence, an infinite sequence of numbers investigated by Leonardo of Pisa, also known as Fibonacci, an Italian mathematician of the thirteenth century.

The sequence is generated using simple addition, and yet it shows up in some unexpected, and complex, ways.

As you read the textbook, you will find interesting areas where these concepts apply.

Objective #1: Find particular terms of a sequence from the general term.

✔ *Solved Problem #1*

1a. Write the first four terms of the sequence whose *n*th term, or general term, is $a_n = 2n + 5$.

$a_n = 2n + 5$

$a_1 = 2(1) + 5 = 7$

$a_2 = 2(2) + 5 = 9$

$a_3 = 2(3) + 5 = 11$

$a_4 = 2(4) + 5 = 13$

The first four terms are 7, 9, 11, and 13.

1b. Write the first four terms of the sequence whose *n*th term, or general term, is $a_n = \frac{(-1)^n}{2^n + 1}$.

$a_n = \frac{(-1)^n}{2^n + 1}$

$a_1 = \frac{(-1)^1}{2^1 + 1} == \frac{-1}{3} - \frac{1}{3}$

$a_2 = \frac{(-1)^2}{2^2 + 1} = \frac{1}{5}$

$a_3 = \frac{(-1)^3}{2^3 + 1} = \frac{-1}{9} = -\frac{1}{9}$

$a_4 = \frac{(-1)^4}{2^4 + 1} = \frac{1}{17}$

The first four terms are $-\frac{1}{3}, \frac{1}{5}, -\frac{1}{9},$ and $\frac{1}{17}$.

Pencil Problem #1

1a. Write the first four terms of the sequence whose *n*th term, or general term, is $a_n = 3n + 2$.

1b. Write the first four terms of the sequence whose *n*th term, or general term, is $a_n = (-1)^n (n + 3)$.

Objective #2: Use recursion formulas.

✔ *Solved Problem #2*

2. Find the first four terms of the sequence in which $a_1 = 3$ and $a_n = 2a_{n-1} + 5$ for $n \geq 2$.

$a_1 = 3$

$a_2 = 2a_1 + 5 = 2(3) + 5 = 11$

$a_3 = 2a_2 + 5 = 2(11) + 5 = 27$

$a_4 = 2a_3 + 5 = 2(27) + 5 = 59$

The first four terms are 3, 11, 27, and 59.

Pencil Problem #2

2. Find the first four terms of the sequence in which $a_1 = 4$ and $a_n = 2a_{n-1} + 3$ for $n \geq 2$.

Objective #3: Use factorial notation.

✔ *Solved Problem #3*

3. Write the first four terms of the sequence whose *n*th term is $a_n = \frac{20}{(n+1)!}$.

$a_n = \frac{20}{(n+1)!}$

$a_1 = \frac{20}{(1+1)!} = \frac{20}{2!} = 10$

$a_2 = \frac{20}{(2+1)!} = \frac{20}{3!} = \frac{20}{6} = \frac{10}{3}$

$a_3 = \frac{20}{(3+1)!} = \frac{20}{4!} = \frac{20}{24} = \frac{5}{6}$

$a_4 = \frac{20}{(4+1)!} = \frac{20}{5!} = \frac{20}{120} = \frac{1}{6}$

The first four terms are $10, \frac{10}{3}, \frac{5}{6},$ and $\frac{1}{6}$.

Pencil Problem #3

3. Write the first four terms of the sequence whose *n*th term is $a_n = \frac{n^2}{n!}$.

Objective #4: Use summation notation.

✔ *Solved Problem #4*

4a. Expand and evaluate the sum: $\sum_{k=3}^{5} \left(2^k - 3\right)$.

$\sum_{k=3}^{5} \left(2^k - 3\right)$

$= \left(2^3 - 3\right) + \left(2^4 - 3\right) + \left(2^5 - 3\right)$

$= (8 - 3) + (16 - 3) + (32 - 3)$

$= 5 + 13 + 29$

$= 47$

Pencil Problem #4

4a. Expand and evaluate the sum: $\sum_{k=1}^{5} k(k+4)$.

4b. Expand and evaluate the sum: $\sum_{i=1}^{5} 4$.

$$\sum_{i=1}^{5} 4 = 4+4+4+4+4$$
$$= 20$$

4b. Expand and evaluate the sum: $\sum_{i=5}^{9} 11$

4c. Express the sum using summation notation. Use 1 as the lower limit of summation and *i* for the index of summation.

$$1^2 + 2^2 + 3^2 + \cdots + 9^2$$

The sum has nine terms, each of the form i^2, starting at $i=1$ and ending at $i=9$.

$$1^2 + 2^2 + 3^2 + \cdots + 9^2 = \sum_{i=1}^{9} i^2$$

4c. Express the sum using summation notation. Use 1 as the lower limit of summation and *i* for the index of summation.

$$2 + 2^2 + 2^3 + \ldots + 2^{11}$$

4d. Express the sum using summation notation. Use 1 as the lower limit of summation and *i* for the index of summation.

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{n-1}}$$

The sum has *n* terms, each of the form $\frac{1}{2^{i-1}}$, starting at $i=1$ and ending at $i=n$.

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{n-1}} = \sum_{i=1}^{n} \frac{1}{2^{i-1}}$$

4d. Express the sum using summation notation. Use 1 as the lower limit of summation and *i* for the index of summation.

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots + \frac{14}{14+1}$$